

Enhanced superconducting proximity effect in clean ferromagnetic domain structures

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We investigate the superconducting proximity effect in a clean magnetic structure consisting of two ferromagnetic layered domains with antiparallel magnetizations in contact with a superconductor. Within the quasiclassical Green's function approach we find that the penetration of the superconducting correlations into the magnetic domains can be enhanced as compared to the corresponding single domain structure. This enhancement depends on an effective exchange field which is determined by the thicknesses and the exchange fields of the two domains. The pair amplitude function oscillates spatially inside each domain with a period inversely proportional to the local exchange field. While the oscillations have a decreasing amplitude with distance inside the domain which is attached to the superconductor, they are enhancing in the other domain and can reach the corresponding normal metal value for a zero effective exchange field. We also find that the corresponding oscillations in the Fermi level proximity density of states as a function of the second domain's thickness has an growing amplitude over a range which depends on the effective exchange field. Our findings can be explained as the result of cancellation of the exchange fields induced phases gained by an electron inside the two domains with antiparallel magnetizations.

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I. INTRODUCTION

In recent years mesoscopic ferromagnet-superconductor (FS) hybrid structures have been studied very extensively.¹ These structures provide the possibility for the controlled studies of the coexistence of the magnetic and superconducting orderings in the presence of the phase coherent effects. The motivation also has come from the interesting potential applications, such as the proposal of the so called π -SFS Josephson junctions² as the solid state qubits.^{3,4} The properties of a normal metal (N) in close contact with an S, find superconducting characteristics due to the penetration of the superconducting correlations (induced order parameter) inside the normal metal. This proximity effect is rather specific in an F metal, in which the interplay between the spin splitting exchange interaction and the singlet superconducting correlations makes the induced superconducting order parameter to have a spatial damped oscillatory behaviour.⁵ One of the manifestations of this effect is that the density of states (DOS) of an F layer in contact to S oscillates with varying the thickness and amplitude of the exchange field of the F layer.^{6,7} The oscillations of the proximity DOS in thin F layers with different thicknesses have been observed in experiments.⁸

The proximity effect in F is rather short range as compared to a normal metal case in which the superconducting correlations can penetrate over the normal coherence length $\xi_N = v_F/T$. Here v_F is the Fermi velocity and we use the units in which $\hbar = k_B = 1$. When the exchange field h is homogeneous, the induced order parameter contains the singlet component and also a triplet component with the zero projection $S_z = 0$ of the total spin.⁹ These components can survive in a clean F only over a short distance $\xi_F = v_F/h$ from the FS interface. In spite of this

observation there have been experiments^{10,11,12} in which the superconducting proximity effect in F are reported which have an amplitude and range much stronger and longer than what the above described theory predicts.

In a pioneering work Bergeret et al.^{9,13,14,15} studied the possibility of a long range proximity in ferromagnets with a local inhomogeneous magnetization. They showed that in the presence of a nonhomogeneous spiral magnetization vector close to the FS interface a triplet component of the superconducting order parameter with projections $S_z = \pm 1$ can be generated in addition to the singlet and the triplet $S_z = 0$ components. These triplet components are not affected by the exchange splitting and can spread over a long distance of the normal coherence length ξ_N . Very recently new experimental evidences of the long range superconducting correlations in ferromagnets have been reported. Sosnin et al.¹⁶ observed the superconducting phase-periodic conductance oscillations of a ferromagnetic Ho wire at a magnetic phase with a conical magnetization vector, in contact with the Al superconducting ring. They found that the phase coherent oscillations sustain for the Ho wire lengths up to 150nm at $T = 0.27\text{K}$, which is much longer than $\xi_F \sim 6\text{nm}$. There has been also reports of observing Josephson supercurrent in SFS contacts with stronger CrO_2 ferromagnetic contacts at even longer distances $0.3 - 1\mu\text{m}$, by group of Klapwijk in Delft.¹⁷ In order to explain such long range effects, they refer to the generation of the Bergeret's triplet component with $S_z = \pm 1$, due to the inhomogeneity of the magnetization.

In order to induce the long range triplet components $S_z = \pm 1$ in F from a singlet superconducting condensate in S, the nonlocal noncollinear orientation of the magnetization vectors is essential. The triplet correlations will not be induced for a collinear configuration of the magnetization vector. In this paper we study the pos-

sibility of a long range superconducting proximity effect in a ferromagnetic structure with collinear magnetization vectors. We consider a clean FS proximity system in which F consists of two layered domains F_1 and F_2 whose magnetization vectors are pointed antiparallel to each other and are separated by a sharp domain wall of negligible width (see Fig. 1). Using the quasiclassical Eilenberger equation,¹⁸ we calculate the superconducting pair amplitude function (order parameter) and the proximity DOS in the structure. The pair amplitude function (PAF) spatially oscillates within each domain with a period which is determined by the amplitude of the local exchange field.

We find that while in F_1 the amplitude of the PAF oscillations decreases with the distance from S, in F_2 it can increase over a distance which depends on an effective exchange field determined by the thicknesses and the exchange fields of F_1 and F_2 . When F_1 and F_2 have the same thicknesses and exchange fields, the PAF oscillations are amplified over whole of F_2 and reaches to the value of the PAF for the corresponding NS structure. We can understand this effect by noting that a quasiparticle travelling through F with a given spin state will gain additional phases in F_1 and F_2 due to the exchange field splittings. Since the exchange fields are oriented antiparallel to each other the phases gained in F_1 and F_2 will have opposite signs, leading to a phase cancellation and suppression of the exchange fields effect. The similar effect was found before in Josephson SFS junctions with F having a domain structure.^{19,20} We further study the effect of this phase cancellation in the local proximity DOS at the Fermi level. We show that it has an oscillatory behaviour with respect to the exchange field and the thickness of F_2 . The oscillations are amplified up to the values of the thickness and exchange field which cancels the phase effect of F_1 completely. At this point the DOS goes to zero corresponding to the normal metal case. This kind of long range penetration differs from the one introduced by Bergeret et al., as it has an oscillatory behaviour and is superposition of the singlet and triplet components with $S_z = 0$.

The rest of this paper is organized as follows. In the next section we introduce our model of the ferromagnetic domain structure (FDS) in contact with an S and present solutions of the Eilenberger equation for the quasiclassical Green's functions. In Sec. III, we calculate the local subgap DOS and PAF. Sec. IV is devoted to the analysis of the PAF and the proximity DOS in terms of the involved parameters. Finally in Sec. V we present the conclusion.

II. THE MODEL AND BASIC EQUATIONS

The system we study is sketched in Fig. 1. An FDS consisting of two layered domains F_1 and F_2 is connected to a superconductor (S) in one side and is bounded on the other side by an insulator or vacuum. $F_{1,2}$ have thick-

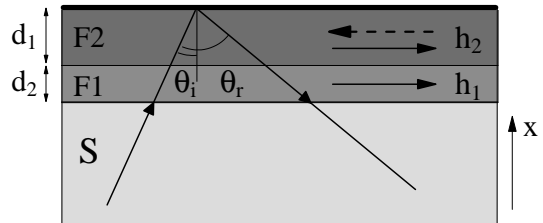


FIG. 1: Schematic of the ferromagnetic domain structure in contact with a superconductor. A classical electronic path is also shown.

nesses d_1 and d_2 , respectively, with magnetization vectors which are oriented antiparallel to each other. We characterize F_1 and F_2 by mean field exchange splittings h_1 and h_2 , respectively, which are included in the Hamiltonian with different signs for antiparallel orientation of the magnetization vectors. The thicknesses $d_{1,2}$ are larger than the Fermi wave length λ_F and smaller than the elastic mean free path ℓ_{imp} , which allows for a quasiclassical description in the clean limit. We apply the collisionless Eilenberger equation,¹⁸ which in the absence of the spin-flip scatterings, is reduced to the following equation for each of $\sigma = \pm 1$ spin directions:

$$\mathbf{v}_F \nabla \hat{G}_\sigma(\omega_n, \mathbf{v}_F, \mathbf{r}) + \left[(\omega_n - i\sigma h(\mathbf{r})) \hat{\tau}_3 + \hat{\Delta}(\mathbf{r}), \hat{G}_\sigma(\omega_n, \mathbf{v}_F, \mathbf{r}) \right] = 0. \quad (2.1)$$

The matrix Green's function for spin σ has the form

$$\hat{G}_\sigma = \begin{pmatrix} g_\sigma & f_\sigma \\ f_\sigma^\dagger & -g_\sigma \end{pmatrix}, \quad (2.2)$$

where g_σ and f_σ are the normal and anomalous Green's functions, respectively, which depend on Matsubara's frequency $\omega_n = (2n + 1)\pi T$ (T is the temperature), the direction of the Fermi velocity \mathbf{v}_F , and the coordinate \mathbf{r} . Here $\hat{\Delta}(\mathbf{r}) = \Delta(\mathbf{r})\hat{\tau}_1$ is the superconducting pair potential matrix (taken to be real) and $\hat{\tau}_i$ ($i = 1, 2, 3$) denote the Pauli matrices. The matrix Green's function obey the normalization condition $\hat{G}_\sigma^2 = \hat{1}$. We assume that the exchange field is homogeneous in $F_{1,2}$ with opposite signs, and vanishes inside S. Inside $F_{1,2}$ we take $\Delta(\mathbf{r}) = 0$. We also neglect the selfconsistent variation of the pair potential close to the F_1 S interface, thus $\Delta(\mathbf{r})$ is constant inside S.

In the ballistic limit we can solve Eq. (2.1) along a classical electronic trajectory. A typical trajectory is shown in Fig. 1. We parameterize the trajectory by a variable $-\infty < \tau < \infty$. An electron coming from bulk of the S, $\tau = -\infty$, enters FDS at F_1 S interface, $\tau = 0$, and subsequently F_2 at F_1 F_2 interface, $\tau = l_{11}/v_F$. The electron undergoes a reflection at the insulator and travels back along the returned segment of the trajectory toward the S. In the returned path the variable τ at F_1 F_2 and F_1 S interfaces is given by $\tau = (l_{11} + l_2)/v_F$ and

$\tau = \tau_0 = (l_1 + l_2)/v_F$, respectively. After passing through F_1S interface, the electron returns to bulk of the S where $\tau = +\infty$. Here l_{11} (l_{12}) is the length of the trajectory segment inside F_1 at the coming (returned) part. The total length of the trajectory inside $F_{1,2}$ are $l_1 = l_{11} + l_{12}$ and l_2 , respectively. To take into account the roughnesses at the insulator surface we consider the reflections to be diffusive. Thus the directions of the coming and returned parts of the trajectory are completely uncorrelated.

The solutions of Eq. (2.1) are restricted by the boundary conditions. These conditions determine the behaviour of the matrix Green's function upon crossing different interfaces in the structure, and also its limiting equilibrium values at bulk of the S. The matrix Green's function approaches to the bulk value $\hat{G}_\sigma(\text{bulk}) = (\omega_n \hat{\tau}_3 + \Delta \hat{\tau}_1)/\Omega_n$,²¹ where $\Omega_n = \sqrt{\omega_n^2 + \Delta^2}$, at the beginning and the end of a trajectory ($\tau = \pm\infty$), and $\Delta(T)$ is the superconducting gap at the temperature T .²² For simplicity we assume that the F_1S and F_1F_2 interfaces are ideally transparent. For F_1F_2 interface our assumption means that the corresponding domain wall is sharp with a negligible width. In this case the boundary conditions are reduced to the continuity of the matrix Green's function at these interfaces.

By imposing the above mentioned boundary conditions on the solutions of Eq. (2.1), we found that along a trajectory the normal Green's function g_σ inside FDS ($0 \leq \tau \leq \tau_0$) does not depend on τ . It depends only on the lengths l_1 and l_2 , and has the form

$$g_\sigma(l_{11}, l_{12}) = \tanh(\beta_{n\sigma} + \alpha_n), \quad (2.3)$$

where $\beta_{n\sigma} = (\omega_{n\sigma}^{(1)} l_1 + \omega_{n\sigma}^{(2)} l_2)/v_F$ and $\alpha_n = \sinh^{-1}(\omega_n/\Delta)$, with $\omega_{n\sigma}^{(j)} = \omega_n - i\sigma h_j$. Inside S, the normal Green's function depends on τ . For the coming segment of the trajectory $\tau \leq 0$, we obtain the following result

$$g_\sigma(\omega_n, \tau, l_{11}, l_{12}) = \tanh \alpha_n + \frac{\sinh(\beta_{n\sigma})}{\cosh \alpha_n \cdot \cosh(\beta_{n\sigma} + \alpha_n)} \cdot e^{2\Omega_n \tau}. \quad (2.4)$$

For the returned segment of the trajectory, $\tau \geq \tau_0$, the normal Green's function is related to the one for incoming segment via $g_\sigma(\omega_n, \tau_0 - \tau, l_{11}, l_{12})$.

For the anomalous Green's function we find the following expressions in different segments of the trajectory

$$f_\sigma(\omega_n, \tau, l_{11}, l_{12}) = \begin{cases} (1 - e^{2\Omega_n \tau})/\cosh \alpha_n + \exp(\beta_{n\sigma} + 2\Omega_n \tau)/\cosh(\beta_{n\sigma} + \alpha_n) & \tau \leq 0 \\ \exp(\beta_{n\sigma} - 2\omega_{n\sigma}^{(1)} \tau)/\cosh(\beta_{n\sigma} + \alpha_n) & 0 \leq \tau \leq l_{11}/v_F \\ \exp[\beta_{n\sigma} + 2(\omega_{n\sigma}^{(2)} - \omega_{n\sigma}^{(1)})l_{11}/v_F - 2\omega_{n\sigma}^{(2)} \tau]/\cosh(\beta_{n\sigma} + \alpha_n) & l_{11}/\tau \leq \tau \leq (l_{11} + l_2)/v_F \\ \exp[-\beta_{n\sigma} + 2\omega_{n\sigma}^{(1)}(\tau_0 - \tau)]/\cosh(\beta_{n\sigma} + \alpha_n) & (l_{11} + l_2)/\tau \leq \tau \leq \tau_0 \\ (1 - e^{2\Omega_n(\tau_0 - \tau)})/\cosh \alpha_n + \exp[-\beta_{n\sigma} + 2\Omega_n(\tau_0 - \tau)]/\cosh(\beta_{n\sigma} + \alpha_n) & \tau \geq \tau_0. \end{cases} \quad (2.5)$$

Eqs. (2.3-2.5) specify the matrix Green's function Eq. (2.2) in all points of the structure. Using these results we can extract all the interested physical quantities. In the next section we use these results to calculate the local electronic DOS and the profile of the superconducting PAF.

III. PROXIMITY DOS AND THE SUPERCONDUCTING PAIR AMPLITUDE FUNCTION

The DOS is expressed in terms of the normal Green's function. To find the DOS per trajectory, we have to calculate

$$N(E, \tau, l_{11}, l_{12}) = \frac{N_0}{2} \sum_{\sigma=\pm 1} \Re g_\sigma(\omega_n = -iE + 0), \quad (3.1)$$

where N_0 is the DOS at the Fermi level in the normal state. In the following we will calculate the proximity DOS for energies below the gap ($|E| \leq \Delta$). Replacing Eq. (2.3) into Eq. (3.1), for the DOS inside the FDS ($0 \leq \tau \leq \tau_0$) we find

$$N(E, l_{11}, l_{12}) = \frac{N_0}{2} \sum_{\sigma=\pm 1} \sum_{n=-\infty}^{+\infty} \pi \delta(k_\sigma^{(1)} l_1 + k_\sigma^{(2)} l_2 - \Phi - n\pi), \quad (3.2)$$

where $k_\sigma^{(i)} = (E + \sigma h_i)/v_F$ and $\Phi = \arccos(E/\Delta)$ is the Andreev phase. From equation (3.2) we can see the fact that the subgap DOS is a sum of δ functions resulting from Andreev bound states of electrons of $E \geq 0$ (positive n 's) and holes of $E < 0$ (negative n 's). The energies also follow from the quasiclassical quantization condition that sets the argument of the δ -function equal to zero. For a given spin direction σ , the phase shift of the An-

dreev states caused by the exchange field has two contributions $\sigma h_1 l_1 / v_F$ and $\sigma h_2 l_2 / v_F$, resulting from the phase gained by a quasiparticle inside F_1 and F_2 , respectively. For antiparallel h_1 and h_2 these phases have opposite signs which can lead to a cancellation of the exchange fields phase effects. As we will see in the following this cancellation is responsible for a long range penetration of the superconducting correlations inside the FDS.

Inside S or $\tau \leq 0$ the subgap DOS is obtained by replacing Eq. (2.4) in Eq. (3.1):

$$N(E, \tau, l_{11}, l_{12}) = \frac{N_0}{2} \pi e^{2\tau \sqrt{\Delta^2 - E^2}} \quad (3.3)$$

$$\times \sum_{\sigma=\pm 1} \sum_{n=-\infty}^{+\infty} \delta(k_\sigma^{(1)} l_1 + k_\sigma^{(2)} l_2 - \Phi - n\pi),$$

where for returned part of the trajectory $\tau \geq \tau_0$, we have the relation $N(E, \tau_0 - \tau, l_{11}, l_{12})$.

The total DOS is obtained by averaging Eqs. (3.2-3.3) over all different possible classical trajectories. This corresponds to an averaging over Fermi velocity directions. To take into account the weak bulk disorder, we include a factor $\exp(-v_F \tau_0 / \ell_{\text{imp}})$ in our averaging.⁷ Denoting the angles between the directions of the coming and returned segments with the normal to the insulator by θ_i and θ_r , respectively, the relations $l_{11} = d_1 / |\cos \theta_i|$, $l_{12} = d_1 / |\cos \theta_r|$ hold. For diffusive reflections θ_i and θ_r are completely uncorrelated, and we make the averaging by an independent integration over these two angles. In this way from Eq. (3.2), inside FDS, we obtain the following result for the total subgap DOS

$$N(E) = \frac{N_0}{2} \sum_{\sigma=\pm 1} \sum_{n=-\infty}^{+\infty} P_{n\sigma}(E) e^{2in \arccos(E/\Delta)}, \quad (3.4)$$

where $P_{n\sigma}(E) = E_2^2(b + 2inA_\sigma)/E_2^2(b)$ with $b = b_1 + b_2$, $b_i = d_i / \ell_{\text{imp}}$ and $E_2(z) = \int_1^\infty du \exp(-zu)/u^2$ is the exponential integral of the second order. Here

$$A_\sigma = k_\sigma^{(1)} d_1 + k_\sigma^{(2)} d_2 = k_\sigma d, \quad (3.5)$$

with $k_\sigma = (E + \sigma h_{\text{eff}})/v_F$, is an effective phase which is determined by the total thickness of FDS $d = d_1 + d_2$ and an effective exchange field defined as

$$h_{\text{eff}} = \frac{h_1 d_1 + h_2 d_2}{d_1 + d_2}. \quad (3.6)$$

From the result given by Eqs. (3.4-3.6) we conclude that the proximity DOS of the FDS is equivalent to an effective single domain ferromagnet with thickness d and the exchange field h_{eff} . A similar result was obtained before for FDS Josephson junctions.¹⁹

By a same averaging over Eq. (3.3) the total subgap DOS is obtained inside S

$$N(E, x) = \frac{N_0}{2} \sum_{\sigma=\pm 1} \sum_{n=-\infty}^{+\infty} Q_{n\sigma}(E, x) e^{2in \arccos(E/\Delta)}, \quad (3.7)$$

where we have introduced the coordinate x along the normal to the interfaces with $x < 0$ and $0 \leq x \leq d$ correspond to the points at the S and FDS sides, respectively. Here $Q_{n\sigma}(E, x) = E_2(b + 2inA_\sigma - 2x\sqrt{\Delta^2 - E^2}/v_F)E_2(b + 2inA_\sigma)/E_2^2(b)$. While the subgap DOS at a given energy E vanishes in bulk of S, it can have a finite value close to F_1S interface, due to the proximity effect. As we found above the proximity DOS in FDS, Eq. (3.4), is constant.

To calculate the PAF we have to do the same averaging over Fermi velocity direction \mathbf{v}_F as we did for the proximity DOS, but this time over the anomalous Green's function given in Eq. (2.5). In fact for PAF normalized in the way that goes to unity at bulk of the S, we have the following relation:¹⁸

$$F(x, T) = \frac{\lambda \pi T}{\Delta} \sum_{\sigma=\pm 1} \sum_n \langle f_\sigma(\omega_n, \tau, l_{11}, l_{12}) \rangle, \quad (3.8)$$

in which $\langle \dots \rangle$ denotes averaging over \mathbf{v}_F and the summation is taken over Matsubara frequencies. Here λ is the electron-phonon interaction constant.

IV. RESULTS AND DISCUSSIONS

Eqs. (3.4, 3.7) and (3.8) express the proximity subgap DOS and the superconducting order parameter in different regions of the FDS proximity system. In the following we analyse the profile of these quantities as well as their dependence on the effective exchange field Eq. (3.6). We pay our attention to the weak ferromagnetic alloys like $\text{Pd}_{1-x}\text{Ni}_x$ with x of the order of 10%, whose exchange field is estimated to be in the range $h = 5 - 20 \text{ meV}$. For a Nb superconductor with $\Delta_0 = \Delta(T = 0) = 1.4 \text{ meV}$ we take $h_{1,2}/\Delta_0$ to be of order 10.⁸

Let us start with analysing the PAF. In Fig. 2 the PAF is plotted versus the coordinate x , for different values of the exchange fields $h_{1,2}$ when the thicknesses $d_1 = d_2 = 0.3\xi_0$ and $T = 0.1T_C$. Here $\xi_0 = v_F/\Delta_0$ is the superconducting coherence length in the clean limit. At the S side close to the F_1S interface the PAF is suppressed with an amplitude which is almost independent of the values of $h_{1,2}$. Inside the FDS the PAF depends strongly on the values of $h_{1,2}$ as well as on their relative signs. While for a fully normal case of $h_1 = h_2 = 0$ (dashed curve) the PAF is exponentially decays with x within the normal coherence length $\xi_N = v_F/T$, it has a quite different behaviour for nonvanishing $h_{1,2}$. For $h_1 = h_2 = 20\Delta_0$ (dotted curve) the FDS is transformed to a single domain ferromagnet, for which PAF oscillates as a function of x with a period $\xi_F = v_F/h$ and a damping amplitude. Here $\xi_F \ll \xi_N$, since $\Delta_0 \ll h$.

An interesting new effect is appeared when the ferromagnetic part is an FDS with the magnetization vectors oriented opposite to each other. In Fig. 2 the PAF is also plotted for the case $h_1 = -h_2 = 20\Delta_0$ (solid line), and in the inset for the cases $h_1 = 12\Delta_0$, $h_2 = -20\Delta_0$ (solid

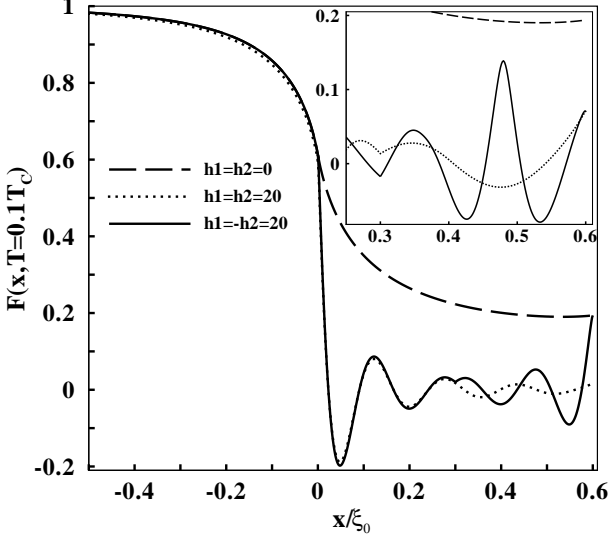


FIG. 2: The pair amplitude function $F(x, T)$ normalized to the bulk value, for the normal metal, single domain ferromagnet and the ferromagnetic domain with $h_{\text{eff}} = 0$ at $T = 0.1T_C$. The thicknesses of domains are taken to be $d_1 = d_2 = 0.3\xi_0$. The exchange fields $h_{1,2}$ are written in the units of Δ_0 . Inset shows the curves for ferromagnetic domain structure with two nonzero effective exchange fields $h_{\text{eff}} = -4\Delta_0$ (solid curve) and $h_{\text{eff}} = 4\Delta_0$ (dotted curve). In the solid curve, the maximum of PAF happens at $x_p = 0.48\xi_0$.

curve) and $h_1 = 20\Delta_0$, $h_2 = -12\Delta_0$ (dotted curve). In these cases the PAF is an oscillatory function in F_1 and F_2 with a period which is given by the local exchange fields h_1 and h_2 . The PAF oscillations are always damping inside F_1 . However the amplitude of the oscillations can enhance in F_2 over a distance from SF_1 interface, depending on the value of h_{eff} . We obtain that the distance at which the PAF reaches its maximum amplitude is given by $x_p = (1 - h_{\text{eff}}/h_2)d$. For the case $h_1 = -h_2 = 20\Delta_0$ the effective exchange field $h_{\text{eff}} = 0$ and hence $x_p = d$, which implies the enhancement of the oscillations amplitude over whole the thickness of F_2 . In this case the phase cancellation effect is complete and the PAF reaches the normal metal value at $x = d$.

For the cases shown in the inset of Fig. 2, the phase cancellation occurs only partially. For $h_1 = 12\Delta_0$, $h_2 = -20\Delta_0$, the effective field $h_{\text{eff}} = -4\Delta_0$ is negative and $x_p = 4/5d < d$. In this case the amplitude of the PAF in F_2 is enhancing for $x < x_p$ and decreasing for $x > x_p$. For $h_1 = 12\Delta_0$, $h_2 = -20\Delta_0$, the effective field $h_{\text{eff}} = 4\Delta_0$ is positive and $x_p = 6/5d > d$. For this case the PAF is enhancing in whole of F_2 . Both the curves, in the inset, are reaching to the same value at $x = d$, which is always smaller than the normal metal value.

We can understand the above mentioned results using the following simple picture. We attribute the proximity effect in FDS to the presence of a macroscopic number

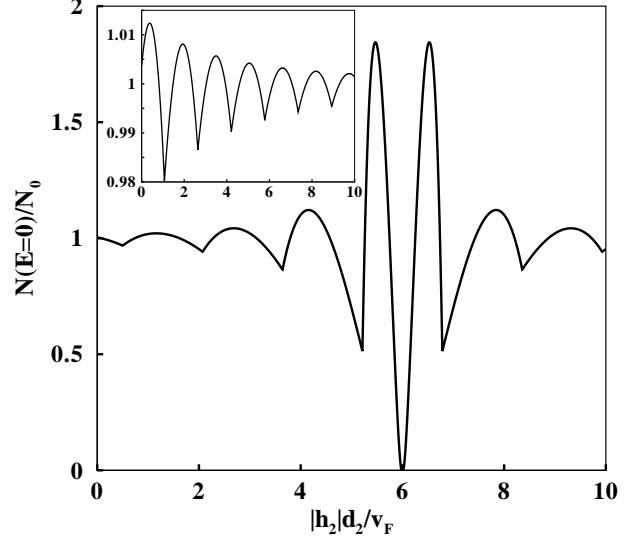


FIG. 3: The behaviour of density of states inside the ferromagnetic domain structure versus the dimensionless variable h_2d_2/v_F , for $E = 0$, $d_1 = 0.3\xi_0$ and $h_1 = 20\Delta_0$. The exchange fields in F_1 and F_2 are antiparallel. Inset: h_1 and h_2 are parallel.

of the Cooper pairs which are escaped from the superconducting condensate of the attached S and enter into the ferromagnetic region. For an homogeneous exchange field in F region the Cooper pairs of electrons with opposite spins gain an additional phase by the exchange splitting induced momentum. The exchange field also tends to break the Cooper pairs by aligning the individual spins of the electrons. These effects, respectively, lead to oscillation and suppression of the Cooper pair wave function, or correspondingly the PAF. In the case of the FDS with antiparallel oriented magnetizations in F_1 and F_2 , there is another alternative possibility. Two electrons of a Cooper pair with opposite spins can be in different domains. A fraction of these Cooper pairs with electrons whose spins are parallel to the local exchange fields $h_{1,2}$, can survive from the above mentioned pair breaking. Such phenomena can lead to an enhancement of the Cooper pair wave function, compared to an homogeneous F.

This effect manifests itself also in the dependence of the proximity DOS of the FDS on h_2d_2/v_F . In Fig. 3 we plot the DOS at the Fermi level ($E = 0$) versus h_2d_2/v_F for $d_1 = 0.3\xi_0$ and $h_1 = 20\Delta_0$. The DOS oscillates around the normal state value N_0 , with a period of $\pi/2$, similar to the DOS oscillations of an homogeneous F.⁶ But now in contrast to the homogeneous F case for which the amplitude of the oscillations is damped with h_2d_2/v_F (see the inset), the amplitude can grow over a range of this quantity. In analogy with the PAF, this range is determined by the effective exchange field h_{eff} . In this case the DOS amplitude enhances up to $h_2d_2/v_F = -6$, where it

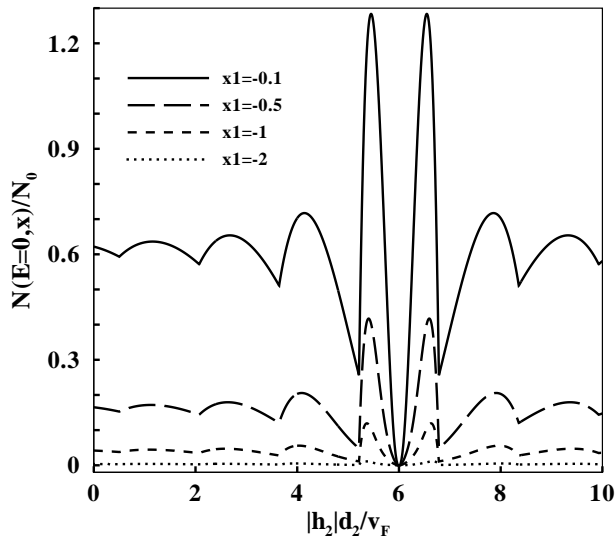


FIG. 4: The behaviour of density of states inside the superconductor in terms of the dimensionless variable $h_2 d_2 / v_F$ for $E = 0$, $d_1 = 0.3\xi_0$ and $h_1 = 20\Delta_0$. $x_1 = x/\xi_0$ and the exchange fields in F_1 and F_2 are antiparallel.

drops to zero corresponding to the normal metal proximity DOS. At this point $h_{\text{eff}} = 0$, which implies a full cancellation of the exchange fields induced phases. We obtain the same dependence of the Fermi level DOS on $h_2 d_2 / v_F$, inside S. This is shown in Fig. 4 for different distances from $F_1 S$ interface x , when $d_1 = 0.3\xi_0$ and $h_1 = 20\Delta_0$. The DOS oscillations with $h_2 d_2 / v_F$ is the same as the DOS of FDS, but now it is around a value which depends on x . Increasing x from zero ($F_1 S$ interface) this value decreases monotonically from N_0 , and approaches zero at the bulk of the S, $x \gtrsim \xi_0$.

V. CONCLUSION

In conclusion we have studied the superconducting proximity effect in a ferromagnetic domain structure

which consists of two clean layered domains $F_{1,2}$ separated by a sharp domain wall. Using the quasiclassical Green's functions approach, we have calculated the pair amplitude function (PAF) and the proximity density of states (DOS). Our results show that in most cases the behaviour of the FDS proximity structure is equivalent to a single domain ferromagnet with thickness $d = d_1 + d_2$ and an effective exchange field $h_{\text{eff}} = (h_1 d_1 + h_2 d_2) / (d_1 + d_2)$. The PAF inside the FDS has spatial oscillations with a period which is inversely proportional to the magnitude of the exchange field in each domain. We have found that while the oscillations in the first domain is always damping with distance from the superconductor, it is enhancing in the second one due to the compensation of the spin splitting effects, by the two oppositely oriented exchange fields. For $h_{\text{eff}} = 0$, the compensation is complete and the amplitude of the oscillations in F_2 reaches to the value of the corresponding normal metal PAF.

We further have found that the exchange splitting compensation has an apparent effect in the oscillatory behaviour of the proximity DOS with the thicknesses of the ferromagnets. Inside the FDS, the Fermi level DOS oscillations around the normal state value has an enhancing amplitude with the thickness of F_2 and vanishes at a thickness for which $h_{\text{eff}} = 0$. Increasing further the thickness of F_2 , the DOS increases back and start to have a damped oscillations around the normal state value. We have explained our results in a simple picture based on surviving of a penetrated Cooper pairs inside FDS, whose two electrons are in different domains with spin directions parallel to the local exchange fields.

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